1. A chicken lays n eggs. Each egg independently does or doesn’t hatch, with probability p of hatching. For each egg that hatches, the chick does or doesn’t survive (independently of the other eggs), with probability s of survival. Let N ⇠ Bin(n, p) be the number of eggs which hatch, X be the number of chicks which survive, and Y be the number of chicks which hatch but don’t survive (so X + Y = N). Find the marginal PMF of X, and the joint PMF of X and Y . Are they independent?

Ans:-

P(X=i)=∑j=in(ji)si(1−s)j−i(nj)pj(1−p)n−j

=∑j=inj!i!(j−i)!n!j!(n−j)!si(1−s)j−ipj(1−p)n−j

=∑j=inn!i!(j−i)!(n−j)!si(1−s)j−ipj(1−p)n−j

=∑r=0n−in!i!r!(n−i−r)!si(1−s)rpr+i(1−p)n−i−r

=n!i!(n−i)!(ps)i(1−p)n−i∑r=0n−i(n−i)!r!(n−i−r)!(1−s)rpr(1−p)−r

=(ni)(ps)i(1−p)n−i∑r=0n−i(n−ir)((1−s)p1−p)r

=(ni)(ps)i(1−p)n−i(1+(1−s)p1−p)n−i

=(ni)(ps)i(1−p)n−i(1−p+p−ps1−p)n−i

=(ni)(ps)i(1−p)n−i(1−ps1−p)n−i=(ni)(ps)i(1−ps)n−i

=Bin(i|n,ps).